UNIT 5: SEQUENCES

Exercise 1: Find the first six terms of the sequences given by the following general terms:

a) $a_n = n^2 - 1$ b) $a_n = \frac{n-1}{n+1}$ c) $a_n = \frac{n}{5}$

d) $a_n = 5n - 2$ e) $a_n = (n+1)^3$ f) $a_n = 2^n - n$

Exercise 2: Find the 17th term in these sequences:

a) $a_n = 5^{n-1}$ b) $a_n = \frac{n(n+3)}{2}$ c) $a_n = n \cdot \pi$

Exercise 3: Find a non-recurring general term a_n in the following sequences:

a) $\{0, 1, 2, 3, 4, 5, \cdots\}$ b) $\{1, 8, 27, 64, 125, 216, \cdots\}$

c) $\{5, 7, 9, 11, 13, 15, \cdots\}$ d) $\{3, 9, 27, 81, 243, 729, \cdots\}$

e) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \cdots\right\}$ f) $\left\{2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}, \frac{7}{36}, \cdots\right\}$

g) $\{-1, 2, -3, 4, -5, 6, \cdots\}$

Exercise 4: Find the general term a_n in the following sequences:

a) $\{12, 14, 16, 18, 20, 22, \cdots\}$ b) $\{1, 1, 2, 3, 5, 8, 13, 21, \cdots\}$

c) $\{0, 1, 0, 1, 0, 1, \cdots\}$ d) $\{1, 5, 9, 13, 17, 21, \cdots\}$

e) $\{2, 6, 18, 54, 162, 486, \cdots\}$ f) $\{7, 5, 3, 1, -1, -3, \cdots\}$

g) $\{5, 8, 11, 14, 17, 20, \cdots\}$ h) $\left\{-\frac{1}{3}, 0, \frac{1}{5}, \frac{2}{6}, \frac{3}{7}, \frac{4}{8}, \cdots\right\}$

i) $\left\{-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, -\frac{1}{25}, \cdots\right\}$ j) $\left\{0, \frac{1}{7}, \frac{4}{49}, \frac{9}{343}, \frac{16}{2401}, \cdots\right\}$

k) $\{9, 5, 4, 1, 3, -2, 5, -7, \cdots\}$ l) $\{3, 2, 6, 12, 72, 864, \cdots\}$

Exercise 5: In the following lists of numbers, find which ones are arithmetic progressions. In that case, indicate the first term, the common difference and the general term:

a) $\{2, 7, 12, 17, 22 \cdots\}$ b) $\{3, 7, 12, 18, 25 \cdots\}$

c) $\{4, 0, -4, -8, -12 \cdots\}$ d) $\{2, 6, 18, 54, 162 \cdots\}$

e) $\{5, 12, 19, 26, 28 \cdots\}$ f) $\{-7, -11, -15, -19, -23 \cdots\}$

Exercise 6: Find the 15th term and the general term of the AP $\{16, 11, 6, 1, -4, -9 \cdots\}$

Exercise 7: The first term of an AP is -3and the 12th term is 41. Determine the common difference

Exercise 8: The common difference of an AP is 5 and the 10th term is 43. Find its first term

Exercise 9: The first term of an AP is -2 and the 11th term is 18. Find its 15th term

Exercise 10: The twelfth term of an AP is 29 and the common difference is 2. Find the general term

Exercise 11: The fifteenth term of an AP is -30 and the first term is 12. Find a_{35}

Exercise 12: The 12th term of an AP is -28 and the 18th term is -46. Find its first term and the common difference

Exercise 13: The 5th term of an AP is 23 and the 12th term is 37. Find the general term and a_{30}

Exercise 14: Which term of the AP $\{5, 2, -1, -4 \cdots\}$ is -22?

Exercise 15: Which term of the AP

a) $\{100, 95, 90, 85, \cdots\}$ is -35?

b) $\left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \cdots\right\}$ is $\frac{25}{4}$?

Exercise 16: Find the sum of the first 12 terms of the APs

a) $\{11, 16, 21, 26, 31, \cdots\}$

b) $\{-151, -148, -145, -142, \cdots\}$

Exercise 17: Find the sum of the first 15 terms of these APs:

a) $\{11, 6, 1, -4, -9, \cdots\}$

b) $\{7, 12, 17, 22, 27, \cdots\}$

Exercise 18: Find the following sum: $2+5+8+11+\cdots+59$

Exercise 19: Find the following sum: $1+5+9+13+\cdots+137$

Exercise 20: Find the sum of all natural numbers between 1 and 1000 which are divisible by 3

Exercise 21: Find the sum of an arithmetic series whose first term is one, the common difference is three, and last term is one hundred.

Exercise 22: In an arithmetic progression we know that $a_{29} = 303$ and the sum of the first twenty-nine terms equals 4321. Find the general term.

Exercise 23: In an arithmetic progression we have that $a_1 = 13$ and $a_{32} = -204$. Find the sum of the first fifty terms.

Exercise 24: How many terms of the AP $\{2, 4, 6, 8, 10, \cdots\}$ are needed to get a sum of 210?

Exercise 25: In a field we have fifty one rows of trees. Each row has two more trees than the previous one. The twenty-sixth row has fifty-seven trees. How many trees do we have in total?

Exercise 26: There are 20 rows of seats on a concert hall: 25 seats are in the 1st row, 27 seats on the 2nd row, 29 seats on the 3rd row, and so on. If the price per ticket is $25, how much will be the total sales for a one-night concert if all seats are taken?

Exercise 27: A person buys a washing machine by installments. The first month he pays €24.4, the second month €28.6, the third €32.8, and so on. The last month he pays €62.2

a) How long was he paying?
b) What's the final price of the washing machine?
c) If the price was 375€, what percentage interest were they charged?

Exercise 28: After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes the first day. Each day thereafter, he suggests that you increase that time by 6 minutes. How many days will it be before you are up to jogging an hour per day?

Exercise 29: A display of cans on a grocery shelf consists of 20 cans on the bottom, 18 cans in the next row, and so on in an arithmetic sequence, until the top row has 4 cans. How many cans, in total, are in the display?

Exercise 30: A projectile fired vertically upward rises 1500 feet in the first second, 1450 feet the following second, 1400 feet the third second, and so on.

a) How many feet does it rise in the 20th second?
b) When does the projectile stop?

Exercise 31: Work out the depth of a well that I want dug if I have paid €76 for the first meter and €15 more than the previous one for each meter afterwards, and I paid a total of €9331.

Exercise 32: Find the first five terms of a GP with initial term $a_1 = 1$ and common ratio $r = \frac{1}{2}$

Exercise 33: Find the 10th and 20th terms of a GP with initial term $a_1 = 3$ and common ratio $r = 2$

Exercise 34: Find the 10th term in the GP with initial term $a_1 = 45$ and common ratio $r = 0.2$

Exercise 35: Find the 7th term in the GP $\{2, 6, 18, 54, \cdots\}$

Exercise 36: Find the 20th term in the GP $\left\{-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \cdots\right\}$

Exercise 37: Find the 17th term in the GP with common ratio $r = 4$ if $a_9 = 196608$

Exercise 38: The tenth term of a GP is 524288, and the first term is 2. Find the general term.

Exercise 39: In a GP, $a_1 = 1$ and $a_{21} = 0.0115292$. Find the general term.

Exercise 40: In a GP, find r given that $a_1 = 10$ and $a_{20} = 10^{-18}$

Exercise 41: In a GP, find the general term given that $a_3 = 20$ and $a_7 = 320$

Exercise 42: In a GP we know that $a_5 = 1250$ and $a_{10} = 3906250$. Find the general term

Exercise 43: In a GP we know that $a_4 = 0.875$ and $a_9 = 0.027344$. Find the general term

Exercise 44: In a GP we know that $a_3 = 81$ and $a_{10} = -3177147$. Find the general term

Exercise 45: How many terms are there in the geometric sequence $\{2, 4, 8, \cdots, 131072\}$

Exercise 46: Find the number of terms in the geometric sequence $\{1, 3, 9, 27, \cdots, 1162261467\}$

Exercise 47: Given the terms $a_1 = \frac{3}{512}$ and $a_6 = \frac{3}{16384}$ of a GP, find the value of a_{30}

Exercise 48: An investor deposits €1500 in a bank account. The bank offers a compound interest rate of 0.7% per year if you don't touch the money. What is the value of the investment 4 years later?

Exercise 49: Each year the annual global demand for olive oil increases by 3%. In 2015, the world olive consumption was of 3295911 tons. What's the expected demand for 2025?

Exercise 50: Find the sum of the first thirty-five terms of a GP with initial term $a_1 = 7$ and common ratio $r = 3$

Exercise 51: Find the sum of the geometric series $128 + 64 + 32 + 16 + \cdots + 1$

Exercise 52: Find the sum of the geometric series $8 - 4 + 2 - 1 \cdots$ if there are sixteen terms in the series

Exercise 53: Find the sum of the GP $\left\{1, \frac{1}{3}, \frac{1}{9}, \cdots\cdots, \frac{1}{177147}\right\}$

Exercise 54: In a GP we know that $a_7 = 364.5$ and $a_{17} = 21523360.5$ Find the general term and the sum of the first twenty-five terms.

Exercise 55: Alex earns a €15000 salary in the first year of his career. Each year, he gets a 2% raise. How much does Alex earn in the first 5 years of his career?

Exercise 56: Nathan opens a savings account that earns 0.08% interest on the last day of the month. His initially deposits €75 on the first day of the first month, and then another €75 on the first day of each month after that. How much money would he have at the end of the first year?

Exercise 57: A ball is dropped onto a hard surface from a height of 2m. Every time it bounces, it rebounds to exactly four fifths of the previous height.

a) Find the general term of the sequence

b) Work out the values of S_{20}, S_{50} and S_{100}

c) Describe what happens to the ball, **interpreting** the previous results.