UNIT 7: EQUATIONS AND SYSTEMS

Exercise 1: Solve the following first degree equations:

a) $4x-3+5x=7-2x-4$

b) $8x-3-4x+2=3x-7+4-x$

c) $x-1-4x=5-3x-6$

d) $7+9x-2-5x=12+4x-9$

e) $5(x+3)=2(x-7)$

f) $4+5(2x-1)=3+2(x-4)$

g) $3x+8(x-5)=11(x-2)$

h) $8x-7(2x-3)=5(4x+1)-26x+16$

Exercise 2: Solve the following equations:

a) $\frac{x-4}{2}+\frac{5x-3}{3}=\frac{1-2x}{4}$

b) $1-\frac{3x-7}{5}=\frac{x}{2}-\frac{5-x}{4}$

c) $\frac{5(x-2)}{8}-x=\frac{1}{3}-\frac{3(1-4x)}{2}$

d) $\frac{2(5-x)}{3}-\frac{5(3x-7)}{6}=\frac{2x-7}{9}$

e) $\frac{5x-9}{45}=\frac{8-3x}{42}$

f) $\frac{3x-7}{2x-3}=\frac{7}{5}$

g) $\frac{12x-5}{x-9}=\frac{1}{2}$

h) $\frac{3(x-7)}{5}=\frac{5(4-3x)}{2}$

Exercise 3: Solve the following quadratic equations:

a) $2x^2-98=0$

b) $3x^2+21x=0$

c) $2x^2-338=0$

d) $18x^2+9x=0$

e) $7x^2-49x=0$

f) $5x^2-70=0$

g) $20x^2-5=0$

h) $x^2=0$

i) $49x^2-9=0$

j) $25x^2-1=0$

Exercise 4: Solve the following second degree equations:

a) $x^2-4x-21=0$

b) $x^2+9x+20=0$

c) $x^2-2x+1=0$

d) $x^2-10x+16=0$

e) $x^2+10x+9=0$

f) $2x^2+2x-24=0$

g) $3x^2+14x-5=0$

h) $x^2+18x+81=0$

i) $x^2-5x+7=0$

j) $9x^2+6x+1=0$

Exercise 5: Solve these equations:

a) $(x-5)^2=0$

b) $(x-3)^2=4$

c) $(x+2)(x-1)=0$

d) $(2x-1)(x-7)=0$

e) $(x+7)^2=1$

f) $(2x-5)^2=0$

Exercise 6: The difference between the square of a number and such number equals the number plus thirty-five. Find said number.

Exercise 7: The width of a rectangular garden is thirteen meters less than the length, and the area is sixty-eight m^2. Find the dimensions of the garden.

Exercise 8: A rectangle is nine as long as it is wide. Its area is 1764 m^2. What are its dimensions?

Exercise 9: In an isosceles triangle the length of the altitude is 9 cm longer than length of the base and the area is $95\,\text{cm}^2$. Find the length of the altitude.

Exercise 10: In a scalene triangle, the length of the base is 12 m less than the length of the altitude and the area is $54\,\text{m}^2$. Find the length of the base.

Exercise 11: Solve the following equations:

a) $(x-3)^2 = 49$

b) $(2x+1)(x-5) = 15x-5$

c) $(3x-5)^2 - 1 = 12x$

d) $(1-2x)^2 + (x-3)^2 = 25$

e) $\dfrac{x+5}{12} = \dfrac{5}{x-2}$

f) $\dfrac{(x-4)^2}{(x+1)^2} = \dfrac{1}{4}$

g) $(3x-1)^2 - (2x+1)(2x-1) = 9x+2$

h) $(x-4)^2 - 8(1-x) = (2x+5)^2$

Exercise 12: Solve:

a) $(x+1)(x-1) + (x-3)^2 = 2(x+1)$

b) $(2x+5)(x-3) = 1-(x+4)^2$

c) $(x-5)^2 - (x+3)(x-3) = (x+9)^2 + 2x + 9$

d) $\dfrac{(x-1)^2}{2} - \dfrac{(x+2)^2}{7} = \dfrac{x-1}{4}$

Exercise 13: Solve these simultaneous equations using the substitution method:

a) $\left.\begin{matrix} x+5y=-4 \\ 5x+2y=3 \end{matrix}\right\}$

b) $\left.\begin{matrix} x-3y=19 \\ 3x-y=25 \end{matrix}\right\}$

c) $\left.\begin{matrix} 2x-3y=1 \\ 5x-y=-4 \end{matrix}\right\}$

d) $\left.\begin{matrix} 5x+y=7 \\ 4x+3y=-1 \end{matrix}\right\}$

e) $\left.\begin{matrix} x+5y=10 \\ 4x-y=-2 \end{matrix}\right\}$

f) $\left.\begin{matrix} x+3y=-14 \\ 3x-6y=3 \end{matrix}\right\}$

Exercise 14: Solve these simultaneous equations using the elimination method:

a) $\left.\begin{matrix} 4x-y=21 \\ 2x+5y=5 \end{matrix}\right\}$ b) $\left.\begin{matrix} 7x-2y=11 \\ x+5y=28 \end{matrix}\right\}$ c) $\left.\begin{matrix} 3x+5y=17 \\ 4x+3y=1 \end{matrix}\right\}$

d) $\left.\begin{matrix} 4x+y=3 \\ 10x+3y=9 \end{matrix}\right\}$ e) $\left.\begin{matrix} 9x+3y=6 \\ 5x+y=4 \end{matrix}\right\}$ f) $\left.\begin{matrix} 2x-4y=9 \\ 3x-6y=5 \end{matrix}\right\}$

Exercise 15: Solve these simultaneous equations using the graphical method:

a) $\left.\begin{matrix} x+y=4 \\ 2x+y=3 \end{matrix}\right\}$ b) $\left.\begin{matrix} x-5y=4 \\ 2x-y=17 \end{matrix}\right\}$ c) $\left.\begin{matrix} 2x-y=3 \\ x-y=5 \end{matrix}\right\}$

d) $\left.\begin{matrix} x+2y=-3 \\ 2x+y=9 \end{matrix}\right\}$ e) $\left.\begin{matrix} 5x+y=7 \\ x+y=5 \end{matrix}\right\}$ f) $\left.\begin{matrix} x+y=4 \\ x-y=7 \end{matrix}\right\}$

Exercise 16: Solve the following systems of equations:

a) $\left.\begin{matrix} \frac{x}{5}+\frac{y}{2}=9 \\ \frac{x}{2}-\frac{y}{7}=3 \end{matrix}\right\}$ b) $\left.\begin{matrix} \frac{2x}{3}+\frac{3y}{5}=7 \\ \frac{4x}{9}+\frac{6y}{5}=\frac{26}{3} \end{matrix}\right\}$

c) $\left.\begin{matrix} \frac{2x}{5}-\frac{3y}{2}=-14 \\ \frac{3x}{5}+\frac{9y}{2}=60 \end{matrix}\right\}$ d) $\left.\begin{matrix} \frac{2x}{3}+\frac{3y}{5}=1 \\ \frac{2x}{7}-\frac{y}{3}=\frac{-59}{7} \end{matrix}\right\}$

Exercise 17: Solve the following simultaneous equations using the indicated method:

a) $\left.\begin{matrix} x-2y=5 \\ 3x-5y=11 \end{matrix}\right\}$ Substitution b) $\left.\begin{matrix} 2x-y=16 \\ 3x+5y=11 \end{matrix}\right\}$ Elimination

c) $\left.\begin{matrix} x+y=1 \\ 3x+y=9 \end{matrix}\right\}$ Graphically d) $\left.\begin{matrix} \frac{x}{8}+\frac{3y}{4}=17 \\ \frac{5x}{4}-\frac{7y}{5}=-8 \end{matrix}\right\}$ Whatever

Exercise 18: Solve and classify the following systems of equations, using the method you prefer:

a) $\left.\begin{matrix} 5x+2y=3 \\ 10x+4y=8 \end{matrix}\right\}$ b) $\left.\begin{matrix} 2x+4y=3 \\ 6x-2y=2 \end{matrix}\right\}$ c) $\left.\begin{matrix} 4x-4y=12 \\ 3x-3y=9 \end{matrix}\right\}$

Exercise 19: The difference of two numbers is ten and the sum of the double of the first number and the second one is seventy-one. Find the numbers.

Exercise 20: In a hostel along the Camino de Santiago we can find double and sextuple rooms. They have a total of 12 rooms and 52 beds. How many rooms of each type are there?

Exercise 21: John has a total of seventeen coins, consisting of fifty cents and one euro coins, which total 11€. How many of each type does he have?

Exercise 22: A man is three times older than his son and the sum of their ages add up to 52 years. How old are they?

Exercise 23: We are going on a field trip to Seville and we want to visit the Cathedral and the Giralda. But we are a lot of people and we cannot enter all at the same time, so we've formed two groups. The first one consists of 23 students and 2 teachers, and they are charged a total of 110€. I will go with the second group, which consists of 30 students 4 teachers, and we are charged a total of 147€. But since I was born in Seville, I don't have to pay the fee to go inside. Lucky me. Knowing that students pay a reduced fee, what are the prices of the tickets for students and adults?

Exercise 24: My animal farm is thriving. After three years I have a total of 197 creatures, consisting on little tamed dragons that only eat mice and fluffy sheep frolicking around all day. And a pony. If I have 45% more sheep than dragons, how many animals of each type live together on my farm?
PS: No animals were hurt during the duration of this problem.

Exercise 25: A woman is now 30 years older than her son. 15 years ago, she was twice as old. What are the present ages of the woman and her son?

Exercise 26: Two and a half kilos of tomatoes and two kilos of peppers cost 6.85€, while three kilos of tomatoes and one and a half kilos of peppers cost 6.6€. What's the price of a kilo of each product?

Exercise 27: Margie is responsible for buying a week's supply of food for the dogs and cats at a local shelter. The food for each dog costs twice as much as those supplies for a cat. She needs to feed 164 cats and 24 dogs. Her budget is €4240. How much can Margie spend on each dog for food?

Exercise 28: The price of olives is 3.6€/kg and the price of gherkins is 2.4€/kg. How many kilos of each product must be used to make 2 kilos of a pickles mix that costs 3.15€/kg?

Exercise 29: How many liters of 25% alcohol solution and how many of 60% alcohol solution must be mixed to produce 35 liters of 50% solution?

Exercise 30: The area of a rectangle gets reduced by 10 square units if its length is reduced by 4 units and its width is increased by 2 units. If we increase the length by 3 units and the width by 4 units, the area is increased by 96 square units. Find the length and width of the rectangle